Solutions to RSPL/2 (DS1)

1. (*a*) Two numbers are 45 and 105

LCM of these numbers is 315

$$\text{HCF} = \frac{\text{Product of the numbers}}{\text{LCM}} = \frac{45 \times 105}{315} = 15$$

$\therefore$ Option (*a*) is correct.

2. (*d*) Here $a = 2q + r$ (Using Euclid's algorithm)

$\therefore$ $0 \leq r < 2$ $(\because b = 2)$

$\therefore$ Possible values of r are 0 and 1.

Option (*d*) is correct.

3. (*a*) 2 and – 3 are zeros of the polynomial

$$p(x) = x^2 + (a + 1)x + b$$

$\therefore$ at $x = 2, p(x) = 0$

$\Rightarrow$ $p(2) = (2)^2 + (a + 1)2 + b$

$0 = 4 + 2a + 2 + b$

$\Rightarrow$ $2a + b = -6$ ...(*i*)

Similarly at $x = -3, p(x) = 0$

$\Rightarrow$ $p(-3) = (-3)^2 + (a + 1)(-3) + b$

$0 = 9 - 3a - 3 + b$

$\Rightarrow$ $-3a + b = -6$ ...(*ii*)

Solving (*i*) and (*ii*)

$\Rightarrow$ $a = 0$ and $b = -6$

$\therefore$ Option (*a*) is correct.

4. (*d*) Here $3x + 4y = 4$ ...(*i*)

and $2x + y = 1$ ...(*ii*)

Solving (*i*) and (*ii*)

add (*i*) and (– 4) × (*ii*)

$3x + 4y = 4$

and $-8x - 4y = -4$

$-5x = 0 \Rightarrow x = 0$

Put $x = 0$ in (*ii*), we get

$2(0) + y = 1 \Rightarrow y = 1$

Now, $2x + 3y = 2(0) + 3(1) = 3$

$\therefore$ Option (*d*) is correct.

5. (*b*) Given numbers aer 3, 5, 5, 7, 7, 7, 9, 9, 9, 9

$$\text{Mean} = \frac{\text{Sum of numbers}}{\text{Total numbers}} = \frac{3+5+5+7+7+7+9+9+9+9}{10}$$

$$= \frac{70}{10} = 7$$

$$\text{Probability of getting mean} = \frac{\text{Number of favourable cases}}{\text{Total number of cases}}$$

$$= \frac{3}{10} = 0.3$$

∴ Option (*b*) is correct

6. (*b*) for acute angle A.

$$\sec A = \sqrt{2}$$

$$\sec A = \sec 45°$$

$$\Rightarrow \quad A = 45°$$

$$\Rightarrow \quad \sin A = \sin 45° = \frac{1}{\sqrt{2}}$$

Option (*b*) is correct.

7. (*c*) Roots of the equations $12x^2 + mx + 5 = 0$ are real and equal

$$\therefore \quad D = 0$$

$$\Rightarrow \quad (m)^2 - 4(12)(5) = 0$$

$$\Rightarrow \quad m^2 - 240 = 0$$

$$m^2 = 240$$

$$m = \sqrt{240}$$

$$= \sqrt{16 \times 15} = 4\sqrt{15}$$

∴ Option (*c*) is correct.

8. (*d*) Given AP is 21, 42, 63, 84 ... 210

Here, $a = 21$

$$d = 42 - 21 = 21$$

Here, $a_n = 210$

$$\Rightarrow \quad a + (n-1)d = 210$$

$$21 + (n-1)21 = 210$$

$$1 + (n-1) = 10$$

$$\Rightarrow \quad n = 10$$

∴ Option (*d*) is correct

9. (*d*)

$$\frac{14587}{1250} = \frac{14587}{2^1 \times 5^4}$$

$$= \frac{14587 \times 2^3}{2^4 \times 5^4} = \frac{14587 \times 2^3}{(10)^4}$$

$$= \frac{116696}{10000} = 11.6696$$

It will terminate after 4 places of decimal.

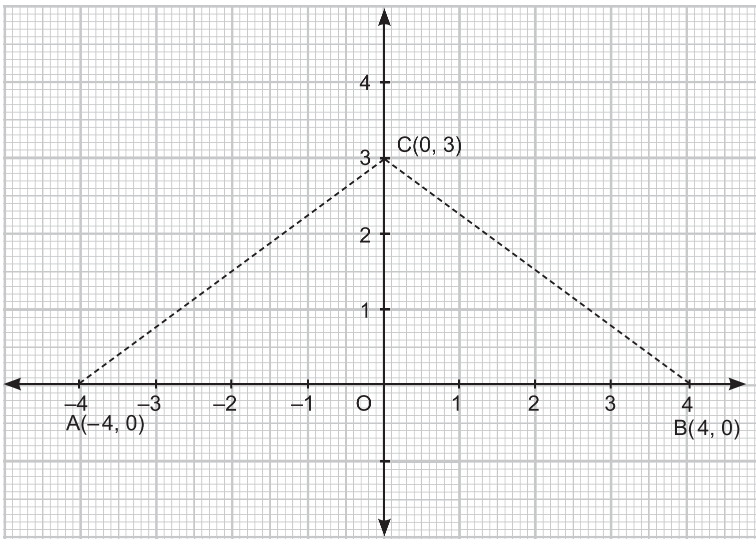
∴ Option (*d*) is correct.

10. (c) Let a_1 be the first term of 1st AP, $\therefore a_1 = -1$ and A_1 be the first term of another AP, $\therefore A_1 = -8$ both AP's have same common difference d.

$$\text{4th term of 1st AP} = a_4 = a + 3d$$
$$\text{4th term of another AP} = A_4 = A_1 + 3d$$
$$a_4 - A_4 = a_1 - A_1 = -1 - (-8) = -1 + 8 = 7$$

$\therefore$ Option (c) is correct.

11. Plot points on graph (coordinate axes)



Here

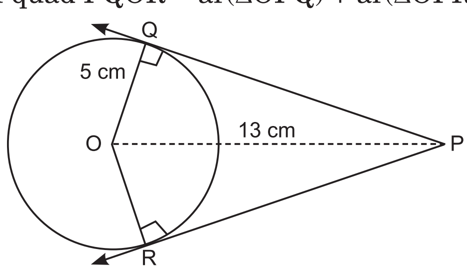
$$AC = BC$$
$$\sqrt{(0+4)^2+(3-0)^2} = \sqrt{(0-4)^2+(3-0)^2}$$
$$\sqrt{16+9} = \sqrt{16+9}$$
$$\sqrt{25} = \sqrt{25} = 5$$

$\therefore$ It forms an isosceles triangle.

12.
$$\sqrt{(4-1)^2+(p-0)^2} = 5$$
$$3^2 + p^2 = 25$$
$$p^2 = 25 - 9 = 16$$
$$p = \pm 4$$

13.
$$PQ = \sqrt{OP^2 - OQ^2} = \sqrt{13^2 - 5^2} = 12$$

Area of quad PQOR = ar(ΔOPQ) + ar(ΔOPR)



$$\text{Area of quadrilateral PQOR} = \frac{1}{2}\times 12\times 5+\frac{1}{2}\times 12\times 5$$
$$= 30 + 30 = 60 \text{ cm}^2$$

14. $$\sin\theta - \cos\theta = 0$$

Squaring both sides

$$(\sin\theta - \cos\theta)^2 = 0$$

$$\sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta = 0$$

$$\sin\theta\cos\theta = \frac{1}{2}$$

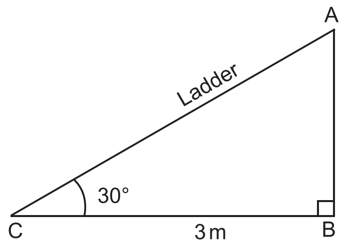
Squaring both sides

$$\sin^2\theta\cos^2\theta = \frac{1}{4}$$

$$\sin^4\theta + \cos^4\theta = (\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta = 1 - \frac{1}{2} = \frac{1}{2}$$

OR

In rt. angled triangle ABC rt. angled at B.



$$\cos 30° = \frac{BC}{AC}$$

$$\frac{\sqrt{3}}{2} = \frac{3}{AC}$$

$$AC = \frac{3 \times 2}{\sqrt{3}} = 2\sqrt{3}\,\text{m}$$

15. Let radius of circle be r cm

and side of square be a cm

Here, circumference of circle $= 2\pi r$

perimeter of square $= 4a$

As per condition $2\pi r = 4a$

$\Rightarrow$ $r = \frac{2a}{\pi}$

Ratio of their areas is

$$\frac{\text{Area of circle}}{\text{Area of square}} = \frac{\pi r^2}{a^2}$$

$$= \frac{\pi}{a^2}\left(\frac{2a}{\pi}\right)^2 = \frac{4a^2\pi}{a^2\pi^2} = \frac{4}{\pi} = 4 : \pi$$

16. The median of the data is 21

Each frequency is increased by 5

Then new median = old median + 5

= 21 + 5 = 26

17. Base diameter of metallic cylinder = 2 cm

Base radius $= \frac{2}{2} = 1$ cm

Height = 16 cm

Let radius of solid sphere = r cm

Volume of 12 solid spheres $= 12\left(\frac{4}{3}\pi r^3\right)$

As per condition

$$\text{Volume of 12 solid spheres} = \text{Volume of metallic cylinder}$$

$$12\left(\frac{4}{3}\pi r^3\right) = \pi(1)^2 \times 16$$

$$16\pi r^3 = 16\pi$$

$$r^3 = 1$$

$\Rightarrow$ $$r = 1$$

$\Rightarrow$ $$d = 2r = 2(1) = 2\text{cm}$$

OR

Let h be the height of the bigger cone then $\frac{h}{2}$ is the height of smaller cone. Let r be the radius of the larger cone and $\frac{r}{2}$ be the radius of smaller cone respectively.

$$\text{Required ratio of volumes} = \frac{\frac{1}{3}\pi\left(\frac{r}{2}\right)^2\frac{h}{2}}{\frac{1}{3}\pi r^2 h} = 1 : 8$$

18. $$\frac{3+5+7 \ldots n \text{ terms}}{5+8+11 \ldots 10 \text{ terms}} = 7$$

$$\frac{\frac{n}{2}[2\times 3+(n-1)2]}{\frac{10}{2}[2\times 5+(10-1)3]} = 7$$

$$\frac{n}{10}\left[\frac{6+2n-2}{10+27}\right] = 7$$

$$2n^2 + 4n = 2590$$

$$2n^2 + 4n - 2590 = 0$$

$\Rightarrow$ $$n^2 + 2n - 1295 = 0$$

$$n^2 + 37n - 35n - 1295 = 0$$

$$n(n + 37) - 35(n + 37) = 0$$

$$(n - 35) = 0 \text{ or } n + 37 = 0$$

$\Rightarrow$ $$n = 35 \text{ or } n = -37 \quad \text{(Rejected)}$$

19. In given figure, $AB \,||\, QR$

$\therefore$ $\angle A = \angle Q$ and $\angle B = \angle R$ (corresponding angle)

$\therefore$ $\Delta PAB \sim \Delta PQR$ (by AA similarity)

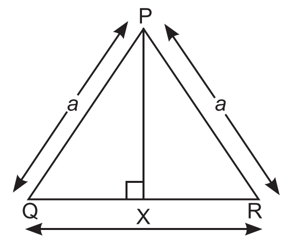
$\Rightarrow$ $$\frac{PA}{PQ} = \frac{AB}{QR} = \frac{PB}{PR}$$

$\Rightarrow$ $$\frac{2.4}{2.4+3.6} = \frac{2}{x}$$

$$\frac{2.4}{6} = \frac{2}{x}$$

$\Rightarrow$ $$x = \frac{2\times 6}{2.4} = 5 \text{ cm}$$

20. In a ΔPQR, PQ = QR = PR = a (say)

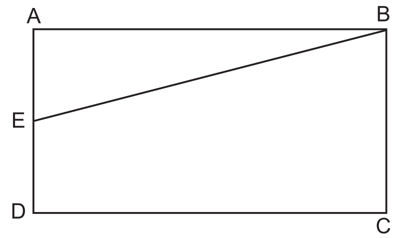


In right triangle PXR, $PR^2 = PX^2 + XR^2$

$$a^2 = PX^2 + \left(\frac{a}{2}\right)^2$$

$$PX^2 = \frac{3a^2}{4}$$

21. In a rectangle ABCD, given E is middle point of AD.



Also given that, AD = 40 cm and AB = 48 cm

$EA = \frac{1}{2}AD$

$\therefore$ EA = 20 cm

So ED = EA = 20 cm

Since ΔABE is a right triangle, right angled at A. Now, by using Pythagoras theorem, we have $EB^2 = EA^2 + AB^2$

$$EB^2 = EA^2 + AB^2$$

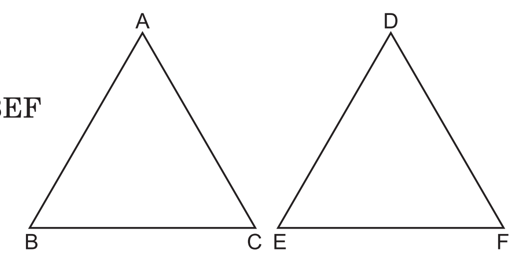
$$EB^2 = 400 + 2304 = 2704$$

$$EB = 52 \text{ cm.}$$

OR

Given that the sides AB and AC and the perimeter P of triangle ABC are respectively three times the corresponding sides DE and DF and the perimeter Q of triangle DEF.

Now , $\frac{\text{Perimeter P}}{\text{Perimeter Q}} = \frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF} = \frac{3}{1}$

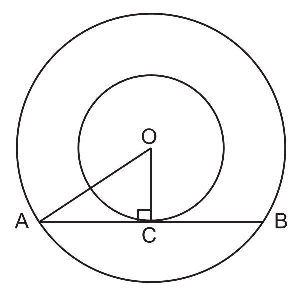


As AB = 3DE, AC = 3DF, BC = 3EF

By SSS similarity, $\Delta ABC \sim \Delta DEF$

Now, $\frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta DEF)} = \frac{AB^2}{DE^2} = \frac{9}{1}$

22. Let OA = r cm and OC = 7 cm and AB = 48 cm



Now $\angle OCA = 90°$, hence OC is perpendicular to AB

$\therefore$ $AC = \frac{1}{2}AB = 24$ cm

Now in right triangle OCA, using Pythagoras theorem, we get

$$OA^2 = OC^2 + AC^2$$

$$r^2 = 7^2 + 24^2$$

$$= 49 + 576 = 625$$

$$r = 25 \text{ cm.}$$

23. Steps of construction:

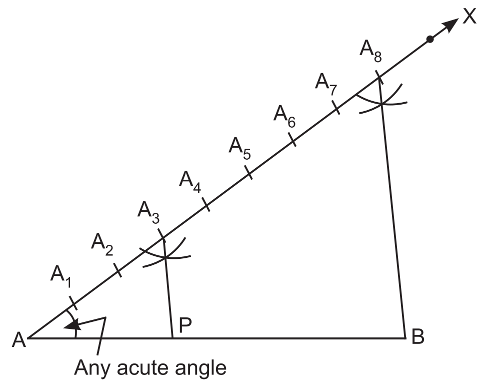
(*a*) Draw AB = 7 cm and darw any ray AX making an acute angle with AB.

(*b*) Locate eight points $A_1, A_2, A_3, ..., A_8$ on $\overrightarrow{AX}$ such that $AA_1 = A_1A_2 = A_2A_3 = ... = A_7A_8$.

(*c*) Join A_8B.

(*d*) At point A_3, draw a line A_3P parallel to A_8B intersecting AB at a point P as shown in figure.

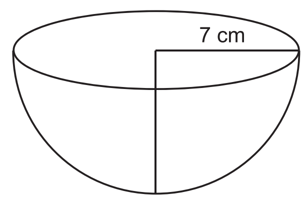
Thus, AP : PB = 3 : 5 is drawn



24. Ram's cap is in the shape of hemisphere having radius 7 cm

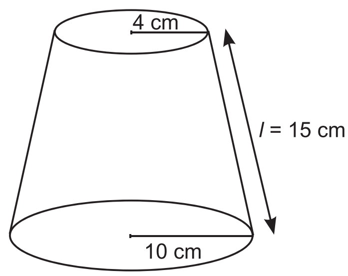
Curved surface area of hemispherical cap = $2\pi r^2$

$$= 2 \times \frac{22}{7} \times 7 \times 7 = 308 \text{ cm}^2$$



Mehmood's cap is in the shape of frustum of a cone.

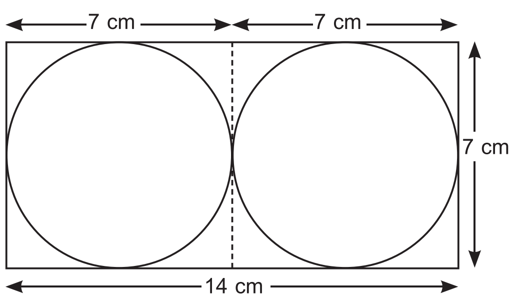
$$r_1 = 4 \text{ cm}$$
$$r_2 = 10 \text{ cm}$$
$$l = 15 \text{ cm}$$



Curved surface area of Mehmood's cap

$$= \pi(r_1 + r_2)l + \pi r_1^2$$
$$= \frac{22}{7}(4 + 10)15 + \frac{22}{7} \times 16$$
$$= \frac{22}{7}[14 \times 15 + 16]$$
$$= \frac{22}{7} \times 226 = 710.28 \text{ cm}^2$$

25.



Let r be the radius of two circles. Let l and b be the length and breadth of the rectangle respectively. Then $r = \frac{7}{2}$ cm, $l = 14$ cm, $b = 7$ cm

Area of the remaining cardboard = Area of rectangle – sum of area of two circles

$$= l \times b - 2\pi r^2$$
$$= 14 \times 7 - 2 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$$
$$= 98 - 77 = 21 \text{ cm}^2$$

OR

Given that the length of minute hand of clock = 14 cm

Radius = 14 cm

Here $\theta = \frac{360}{60} \times 5 = 30°$ (angle made by minute hand in 5 minutes)

Area swept by minute hand in 5 minutes $= \frac{\pi r^2 \theta}{360}$

$$= \frac{22}{7} \times 14 \times 14 \times \frac{30}{360}$$

$$= \frac{22 \times 2 \times 7}{6} = \frac{154}{3} = 51.3 \text{ cm}^2$$

26. Total balls = Number of (red balls + white balls + black balls)

$\therefore \quad x + 2x + 3x = 24$

$\Rightarrow \quad x = 4$

Red balls = 4, white balls = 2 × 4 = 8, black balls = 3 × 4 = 12

(*i*) Probability of getting (not red ball) $= \frac{20}{24} = \frac{5}{6}$

(*ii*) Probability of getting (white ball) $= \frac{8}{24} = \frac{1}{3}$

27. Let us assume that $3 + 2\sqrt{3}$ is a rational number.

Then, there exists integers a and $b (b \neq 0)$ such that

$$3 + 2\sqrt{3} = \frac{a}{b}$$

$$2\sqrt{3} = \frac{a}{b} - 3$$

$$\sqrt{3} = \frac{a - 3b}{2b}$$

Since a, b, 2 and 3 are integers, so $\frac{a-3b}{2b}$ is rational. This shows $\sqrt{3}$ is also rational.

But this is a contradiction as $\sqrt{3}$ is irrational.

Hence our assumption that $3 + 2\sqrt{3}$ is rational number is wrong. Therefore $3 + 2\sqrt{3}$ is an irrational number.

28. Let $f(x) = ax^3 + 3x^2 - bx - 6$

– 1 and – 2 are the zeros of the cubic polynomial $f(x)$

So, $\quad f(-1) = 0$ and $f(-2) = 0$

$\Rightarrow f(-1) = a(-1)^3 + 3(-1)^2 - b(-1) - 6 = -a + b - 3 = 0$ ...(*i*)

$\Rightarrow f(-2) = a(-2)^3 + 3(-2)^2 - b(-2) - 6 = -8a + 2b + 6 = 0$ ...(*ii*)

(– 2)(*i*) + (*ii*), we get

$$2a - 2b = -6$$

$$-8a + 2b = -6$$

$$\overline{\quad -6a = -12 \quad}$$

$\Rightarrow \quad a = 2$

Putting in eq (i), we get

$$-2 + b = 3 \Rightarrow b = 5$$

We get $a = 2$ and $b = 5$.

$$\therefore \quad f(x) = 2x^3 + 3x^2 - 5x - 6$$

$(x + 1)$ and $(x + 2)$ are factors of $f(x)$

$\therefore$ Dividing $f(x)$ by $(x + 1)(x + 2)$

$$\begin{array}{r|l} x^2+3x+2 \,\big)\; 2x^3 + 3x^2 - 5x - 6 & 2x - 3 \\ 2x^3 + 6x^2 + 4x & \\ - \quad - \quad - & \\ \hline -3x^2 - 9x - 6 & \\ -3x^2 - 9x - 6 & \\ + \quad + \quad + & \\ \hline 0 & \end{array}$$

For third zero,

Put $\quad 2x - 3 = 0$

$$\Rightarrow \quad x = \frac{3}{2}$$

OR

Let $p(x) = 3x^3 + 4x^2 + 5x - 13$, $q(x) = 3x + 10$ and $r(x) = 16x - 43$

By division algorithm, we have

$$\text{Dividend} = \text{Divisor} \times \text{quotient} + \text{Remainder}$$

$$\Rightarrow \quad p(x) = g(x) \times q(x) + r(x)$$

$$\Rightarrow \quad 3x^3 + 4x^2 + 5x - 13 = g(x).(3x + 10) + 16x - 43$$

$$\Rightarrow \quad 3x^3 + 4x^2 + 5x - 13 - 16x + 43 = g(x).(3x + 10)$$

$$3x^3 + 4x^2 - 11x + 30 = g(x) \times (3x + 10)$$

$$\Rightarrow \quad g(x) = \frac{3x^3 + 4x^2 - 11x + 30}{3x + 10}$$

$$\begin{array}{r|l} 3x+10 \,\big)\; 3x^3 + 4x^2 - 11x + 30 & x^2 - 2x + 3 \\ 3x^3 + 10x^2 \qquad\qquad & \\ - \quad - \qquad\qquad & \\ \hline -6x^2 - 11x + 30 & \\ -6x^2 - 20x \qquad & \\ + \quad + \qquad & \\ \hline 9x + 30 & \\ 9x + 30 & \\ - \quad - & \\ \hline 0 & \end{array}$$

$\therefore$

29. Let the denominator of the fraction = x

The numerator = $(x - 3)$

The fraction = $\frac{x-3}{x}$

When 2 is added to both the numerator and denominator, then the new fraction

$$= \frac{(x-3)+2}{x+2} = \frac{x-1}{x+2}$$

Now, given that the sum of the original fraction and the new fraction is $\frac{29}{20}$

$$\frac{x-3}{x} + \frac{x-1}{x+2} = \frac{29}{20}$$

$$\frac{(x-3)(x+2)+(x-1)(x)}{x(x+2)} = \frac{29}{20}$$

$$\frac{x^2 - 3x + 2x - 6 + x^2 - x}{x^2 + 2x} = \frac{29}{20}$$

$$\frac{2x^2 - 2x - 6}{x^2 + 2x} = \frac{29}{20}$$

$$40x^2 - 40x - 120 = 29x^2 + 58x$$

$$11x^2 - 98x - 120 = 0$$

$$11x^2 - 110x + 12x - 120 = 0$$

$$11x(x - 10) + 12(x - 10) = 0$$

$$(11x + 12)(x - 10) = 0$$

$$x = 10, \frac{-12}{11}$$

$\frac{-12}{11}$ is rejected, $x = 10$

$$\text{Fraction} = \frac{7}{10}$$

OR

Let the speed of the stream is x km/hr

Total distance to be covered = 30 km

Speed of the boat when it goes 30 km upstream = $(15 - x)$ km/hr

Speed of the boat when it goes 30 km downstream = $(15 + x)$ km/hr

Time taken by the boat to cover given distance upstream = $\frac{30}{15-x}$

Time taken by the boat to cover given distance downstream = $\frac{30}{15+x}$

According to the question, we have

$$\frac{30}{15-x} + \frac{30}{15+x} = 4\frac{1}{2}$$

$$\frac{30(15+x)+30(15-x)}{225-x^2} = \frac{9}{2}$$

$$\frac{450+30x+450-30x}{225-x^2} = \frac{9}{2}$$

$$\frac{900}{225-x^2} = \frac{9}{2}$$

$$9x^2 = 225$$

$$x^2 = 25$$

$$x = \pm 5$$

$x = -5$ is rejected as speed of the stream cannot be negative. Hence speed of the stream is 5 km/hr.

30. Steps of construction:

(*a*) Draw a circle C with centre O and radius 3.5 cm.

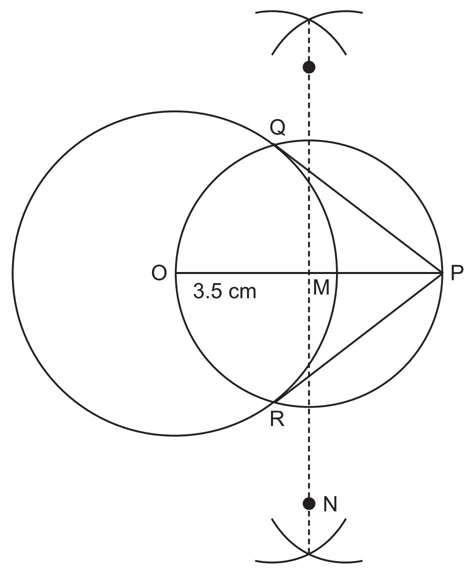
(*b*) Take a point P at a distance of 6.2 cm from its centre.

(*c*) Bisect the line segment OP. Let the point of bisection be M.

(*d*) Taking M as centre and OM as radius, draw a circle which intersects the given circle C at the points Q and R.

(*e*) Join PQ and PR.

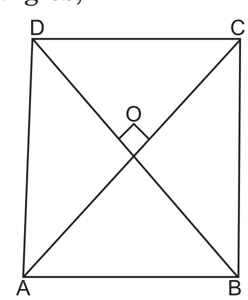
These are the required tangents.



31. Given: ABCD is a rhombus

To Prove: $AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$

Proof: Since the diagonals of a rhombus bisect each other at right angles,



$\therefore$ AC and BD bisect each other at 90°

$\angle AOB = \angle BOC = \angle COD = \angle DOA = 90°$

and AO = CO, BO = OD

Since, AOB is right triangle, right angled at O,

$$\Rightarrow \quad AB^2 = OA^2 + OB^2 = \frac{AC^2}{4} + \frac{BD^2}{4}$$

$$4AB^2 = AC^2 + BD^2 \qquad ...(i)$$

Similarly, we have

$$4BC^2 = AC^2 + BD^2 \qquad ...(ii)$$

$$4CD^2 = AC^2 + BD^2 \qquad ...(iii)$$

$$4AD^2 = AC^2 + BD^2 \qquad ...(iv)$$

Adding all these results, we get

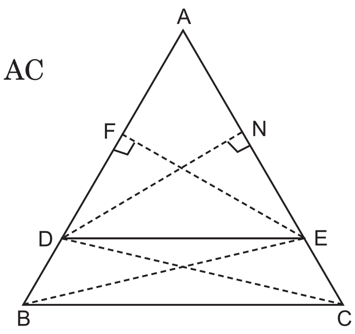
$$4(AB^2 + BC^2 + CD^2 + DA^2) = 4(AC^2 + BD^2)$$ Hence proved.

$$AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$$

OR

Given: In ΔABC, line DE parallel to BC intersects AB at D and AC at E.

To prove: $\frac{AD}{DB} = \frac{AE}{EC}$



Construction: Join BE, CD and draw EF $\perp$ AB and DN $\perp$ AC

Proof: We have

$$\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{\frac{1}{2}\times AD \times EF}{\frac{1}{2}\times BD \times EF} = \frac{AD}{BD} \quad ...(i)$$

Similarly,

$$\frac{ar(\Delta ADE)}{ar(\Delta CDE)} = \frac{\frac{1}{2}\times AE \times DN}{\frac{1}{2}\times EC \times DN} = \frac{AE}{EC} \quad ...(ii)$$

But $\quad ar(\Delta BDE) = ar(\Delta CDE) \quad ...(iii)$

(triangles drawn on the same base between two parallel lines are equal in areas)

Equating (*i*), (*ii*) and (*iii*) we get

$$\frac{AD}{DB} = \frac{AE}{EC}$$

32. LHS $\quad m^2 - n^2 = (\sin\theta + \tan\theta)^2 - (\tan\theta - \sin\theta)^2$

$$= (\tan^2\theta + \sin^2\theta + 2\tan\theta\sin\theta) - (\tan^2\theta + \sin^2\theta - 2\tan\theta\sin\theta)$$

$$= 4\tan\theta\sin\theta$$

RHS $\quad 4\sqrt{mn} = 4\sqrt{(\tan\theta + \sin\theta)(\tan\theta - \sin\theta)}$

$$= 4\sqrt{\tan^2\theta - \sin^2\theta - \tan\theta\sin\theta + \tan\theta\sin\theta}$$

$$= 4\sqrt{\tan^2\theta - \sin^2\theta}$$

$$= 4\sqrt{\frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta}$$

$$= 4\sqrt{\frac{\sin^2\theta - \sin^2\theta\cos^2\theta}{\cos^2\theta}}$$

$$= 4\sqrt{\frac{\sin^2\theta(1 - \cos^2\theta)}{\cos^2\theta}}$$

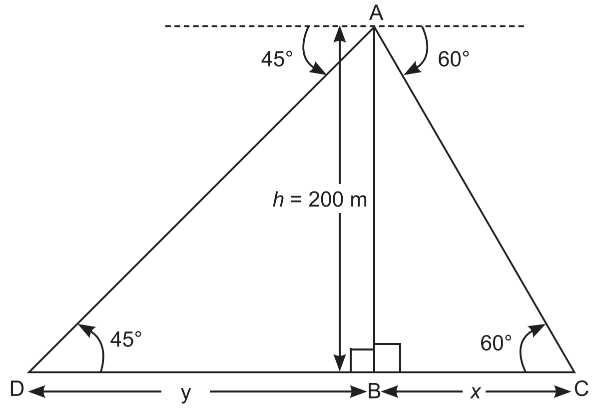
$$= 4\sqrt{\tan^2\theta\sin^2\theta}$$

$$= 4\tan\theta\sin\theta$$

33. Let A be the position of the aeroplane and let C and D be two points on the opposite banks of the river such that the angles of depression at C and D are 60° and 45° respectively.

Let BC = x metres and BD = y metres

We have to find CD i.e $(x + y)$ m



In right triangle ABC,

$$\frac{AB}{BC} = \tan 60°$$

$$\Rightarrow \quad \frac{200}{x} = \sqrt{3}$$

In right triangle ABD,

$$\frac{AB}{BD} = \tan 45°$$

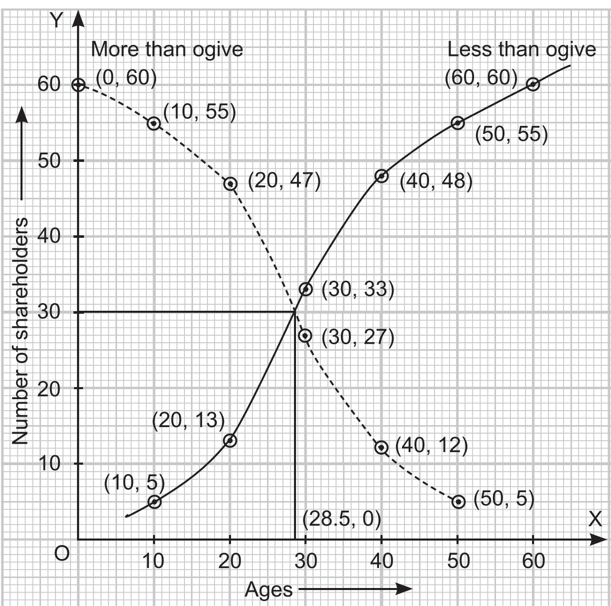
$$\frac{200}{y} = 1$$

$$y = 200 \text{ m}$$

$$CD = x + y = \frac{200}{\sqrt{3}} + 200 = 200\left(\frac{1+\sqrt{3}}{\sqrt{3}}\right) \text{m}$$

34. (*i*)

Age less than	Number of share holders	Age more than	Number of shareholders
10	5	0	60
20	13	10	55
30	33	20	47
40	48	30	27
50	55	40	12
60	60	50	5



Median is 28.5.

(*ii*)

$$\text{Mode} = 3 \text{ median} - 2\text{mean}$$
$$27 = 3(28.5) - 2(\text{mean})$$
$$\text{Mean} = \frac{85.5 - 27}{2} = 29.25$$

35. Let the four numbers of the AP are $a - 3d$, $a - d$, $a + d$ and $a + 3d$ respectively.

$\therefore \quad a - 3d + a - d + a + d + a + 3d = 32$

$\therefore \quad a = 8$

Also $$\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{7}{15}$$
$$\frac{a^2 - 9d^2}{a^2 - d^2} = \frac{7}{15}$$
$$8a^2 = 128\, d^2$$
$$8 \times a^2 = 128\, d^2$$
$$d^2 = \frac{8a^2}{128} = \frac{8 \times 8 \times 8}{128} = 4$$
$$d = \pm 2$$

When $a = 8$ and $d = 2$

$$a - 3d = 2$$
$$a - d = 6$$
$$a + d = 10$$
$$a + 3d = 14$$

When $a = 8$ and $d = -2$

$$a - 3d = 14$$
$$a - d = 10$$
$$a + d = 6$$
$$a + 3d = 2$$

Therefore, four consecutive terms of the AP's are 2, 6, 10, 14 and 14, 10, 6, 2

OR

The houses in a row are numbered consecutively 1, 2, 3, 4,.........49

Here, $$a = 1 \text{ and } d = 1$$
$$S_{X-1} = S_{49} - S_X$$
$$\frac{X-1}{2}[2a + (X - 1 - 1)d] = \frac{49}{2}[2a + (49 - 1)d] - \frac{X}{2}[2a + (X - 1)d]$$
$$\frac{X-1}{2}[2a + (X - 2)d] = \frac{49}{2}[2a + 48d] - \frac{X}{2}[2a + (X - 1)d]$$

Put $a = 1$ and $d = 1$ throughout, we get

$$\frac{X-1}{2}[2 + (X - 2)] = \frac{49}{2}[2 + 48] - \frac{X}{2}[2 + (X - 1)]$$
$$\frac{X^2 - X}{2} = \left(\frac{2450}{2}\right) - \left(\frac{X^2 + X}{2}\right)$$

$$X^2 - X = 2450 - X^2 - X$$
$$2X^2 = 2450$$
$$X^2 = 1225$$
$$X = \pm 35$$

But X cannot be negative $\therefore$ X = 35.

36.

Classes	**Mid values**	$u_i = \frac{x_i - a}{h}$	f_i	$f_i u_i$
0 – 20	10	–3	5	–15
20 – 40	30	–2	x	$-2x$
40 – 60	50	–1	10	–10
60 – 80	70	0	y	0
80 – 100	90	1	7	7
100 – 120	110	2	8	16
			$x + y + 30 = 50$	$-2 - 2x$

$a = 70$, $h = 20$

$$\bar{u} = \frac{\Sigma f_i u_i}{\Sigma f_i}$$
$$\text{Mean} = a + h\bar{u}$$
$$62.8 = 70 + 20\left[\frac{\Sigma f_i u_i}{\Sigma f_i}\right]$$
$$-7.2 = 20 \times \left[\frac{-2 - 2x}{50}\right]$$
$$7.2 = \frac{2}{5}[2 + 2x]$$
$$36 = 4 + 4x$$
$$32 = 4x$$
$$x = 8$$

Also, $x + y + 30 = 50 \Rightarrow 8 + y + 30 = 50 \Rightarrow y = 12$

37. The total surface area of the cube = $6 \times (\text{Edge})^2 = 6 \times 5 \times 5$ sq.cm = 150 sq.cm .

The part of the cube where the hemisphere is attached is not included in the surface area.

So, the surface area of the block = TSA of cube – base area of hemisphere + CSA of hemisphere

$$= 150 - (\pi r^2) + 2\pi r^2$$
$$= 150 + \pi r^2$$
$$= 150 + \frac{22}{7} \times 2.1 \times 2.1$$
$$= (150 + 13.86) \text{ sq.cm} = 163.86 \text{ sq.cm}$$

Cost of painting per sq. cm = ₹ 2

Total cost = 163.86 × 2 = ₹ 327.72

38. It is given that the angles of depression at C and D are 30° and 45° respectively.

Let the speed of the car be x m/minute

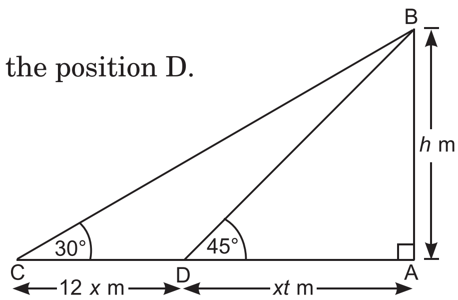
Now, distance covered by the car in 12 minutes is CD

$$\Rightarrow \quad CD = 12 \times x \text{ meters}$$

i.e. $$CD = 12x \text{ meters}$$

Let car takes t minutes to reach the tower AB from the position D.

Then distance AD = xt meters.



In right triangle DAB,

$$\frac{AB}{AD} = \tan 45°$$

$$\Rightarrow \quad \frac{h}{xt} = 1$$

$$\Rightarrow \quad h = xt \qquad ...(i)$$

In right triangle CAB,

$$\frac{AB}{AC} = \tan 30°$$

$$\Rightarrow \quad \frac{h}{xt + 12x} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \sqrt{3}h = xt + 12x \qquad ...(ii)$$

Substituting the value of h from equation (i) in equation (ii), we get

$$\sqrt{3}xt = xt + 12x$$

$$\Rightarrow \quad \sqrt{3}t = t + 12$$

$$\Rightarrow \quad (\sqrt{3} - 1)t = 12$$

$$\Rightarrow \quad t = \frac{12}{\sqrt{3} - 1} = \frac{12}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$$

$$\Rightarrow \quad t = 6(\sqrt{3} + 1) \text{ minutes} = (6 \times 2.732) \text{ minutes}$$

$$(\because \sqrt{3} = 1.732)$$

$$= 16.392 \text{ minutes}$$

Now 0.39 minutes = 0.39 × 60 = 23.40 seconds

$$t = 16 \text{ minutes } 23 \text{ seconds}$$

Thus the car will reach the tower from 'D' in 16 minutes 23 seconds.

39. Let $y = \frac{2x + 3}{x - 3}$

$$\therefore \quad \frac{x - 3}{2x + 3} = \frac{1}{y}$$

The given quadratic equations becomes

$$2y - \frac{25}{y} = 5$$

$$2y^2 - 5y - 25 = 0$$

$$2y^2 - 10y + 5y - 25 = 0$$

$$2y(y-5)+5(y-5) = 0$$
$$(y-5)(2y+5) = 0$$
$$y = 5, -\frac{5}{2}$$

When $\dfrac{2x+3}{x-3} = 5$

$$(2x+3) = 5(x-3)$$
$$2x+3 = 5x-15$$
$$18 = 3x$$
$$x = 6$$

When $\dfrac{2x+3}{x-3} = \dfrac{-5}{2}$

$$4x+6 = -5x+15$$
$$9x = 9$$
$$x = 1$$

OR

Let the time taken by first pipe alone to fill the cistern = x minutes

Time taken by second pipe to fill the cistern = $x + 5$ minutes

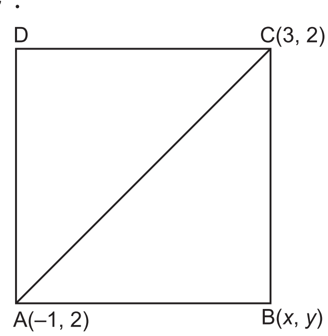
$$\frac{1}{x}+\frac{1}{x+5} = \frac{1}{6}$$
$$\frac{2x+5}{x^2+5x} = \frac{1}{6}$$
$$6(2x+5) = x^2+5x$$
$$x^2-7x-30 = 1$$
$$(x+3)(x-10) = 0$$
$$x = -3, 10$$

$x = -3$ is rejected as time cannot be negative.

$\Rightarrow \quad x = 10.$

Hence, the first pipe takes 10 minutes and the second pipe takes 15 minutes to fill the cistern.

40. Let ABCD be a square and let A(– 1, 2) and C(3, 2) be the given opposite vertices. Let B(x, y) be the unknown vertex. Then AB = BC gives $AB^2 = BC^2$.



So, $(x+1)^2+(y-2)^2 = (x-3)^2+(y-2)^2$

$$x^2+1+2x+y^2+4-4y = x^2+9-6x+y^2+4-4y$$

or $2x+1 = -6x+9$

or $8x = 8$

or $x = 1$ ...(i)

In ΔABC, we have: $AB^2+BC^2 = AC^2$

So, $(x + 1)^2 + (y - 2)^2 + (x - 3)^2 + (y - 2)^2 = (3 + 1)^2 + (2 - 2)^2$

or $2x^2 + 2y^2 + 2x - 4y - 6x - 4y + 1 + 4 + 9 + 4 = 16$

or $2x^2 + 2y^2 - 4x - 8y + 2 = 0$

or $x^2 + y^2 - 2x - 4y + 1 = 0$ ...(*ii*)

Substituting the value of x from (*i*) in (*ii*), we get

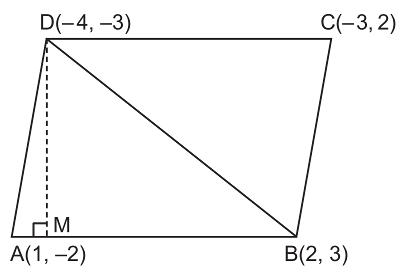
$$1 + y^2 - 2 - 4y + 1 = 0$$

or $y^2 - 4y = 0$ or $y(y - 4) = 0$

i.e., $y = 0$ or $y = 4$

Hence, the required vertices of the square are (1, 0) and (1, 4).

OR



Given vertices of parallelogram are A(1, – 2), B(2, 3), C(–3, 2) and D(–4, –3).

Let DM be the height of the parallelogram.

To find DM, first we find area of triangle DAB.

We know that the area of triangle, with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by

$$\Delta = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

$\therefore$ Area of ΔDAB with vertices D(–4, – 3), A(1, – 2) and B(2, 3) is given by

$$\text{Area of } \Delta DAB = \frac{1}{2}|-4(-2-3) + 1(3-(-3)) + 2(-3+2)|$$

$$= \frac{1}{2}|-4(-5) + 1(6) + 2(-1)|$$

$$= \frac{1}{2}|20 + 6 - 2|$$

$\therefore$ $$\text{Area of } \Delta DAB = \frac{1}{2}|24| = \frac{24}{2} = 12 \text{ sq. units} \quad ...(i)$$

Also, we know that,

$$\text{Area of triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$

$\therefore$ $$\text{Area of } \Delta DAB = \frac{1}{2} \times AB \times DM \quad ...(ii)$$

Using distance formula, length of AB is given by,

$$AB = \sqrt{(2-1)^2 + (3-(-2))^2} = \sqrt{(2-1)^2 + (3+2)^2}$$

$$= \sqrt{(1)^2 + (5)^2} = \sqrt{1 + 25}$$

$$AB = \sqrt{26}$$

$\therefore$ From (*i*) and (*ii*), we get

$$\frac{1}{2} \times \sqrt{26} \times DM = 12$$

$$\Rightarrow \qquad DM = \frac{24}{\sqrt{26}}$$

$$\Rightarrow \qquad DM = \frac{24\sqrt{26}}{\sqrt{26} \times \sqrt{26}} = \frac{24\sqrt{26}}{26}$$

$$\Rightarrow \qquad DM = \frac{12\sqrt{26}}{13} \text{ units}$$

Hence height of the parallelogram is $\frac{12\sqrt{26}}{13}$ units.